On the Theory of the Moist-bulb Hygrometer. *By* JAMES APJOHN, Esq. M.D. M.R.I.A., *Professor of Chemistry in the Royal College of Surgeons.*

(Continued.)

Read April 27, 1835.

At the meeting of the Academy held in November last, I was permitted to read a short memoir on the subject of a formula, at which I had a considerable time previously arrived, for inferring the Dew-point from the indications of the Moist-bulb Hygrometer. This formula was deduced altogether from general considerations ; and, though satisfied, from some hasty observations of my own, that it represented facts with considerable accuracy, I was not, at the time, in possession of evidence which could be considered as establishing this important point in an unequivocal manner. The table which is subjoined to my paper undoubtedly shows, that, within certain limits, my formula is in accordance with experiment ; but the observed depressions in the table are, generally speaking, so small, that a formula in itself incorrect might, it must be admitted, yield results which would deviate from the observed dew-points by quantities not exceeding the possible errors of observation. Berzelius, for example, states (Traité de Chimie, tom. viii. p. 254,) that, from the experiments of August, Bohnenberger, and others, it appeared that the temperature of a thermometer with moistened bulb was an arithmetic mean between that of the air and the dew-point ; and this rule, which would make $t'' = 2t' - t$, though utterly erroneous, would apply to the table appended to my paper, nearly as well as the formula I have deduced. The validity, therefore, of my method required to be more rigorously tested ; and having been for some time engaged in experimental researches, instituted with this object, which have led to interesting, and to me most satisfactory results, I am anxious to submit them, with as little delay as possible, to the judgment of the Academy.

The equation which, as I believe, comprehends the theory of the wet-bulb hygro-meter, is as follows :—

$$f'' = f' - m\,d \times \frac{p}{30}$$

in which f'' is the tension of steam at the dew-point, f' its tension at the temperature of the hygrometer, d the depression or difference between the temperature of the hygrometer and air, p the existing, and 30 the mean pressure, and m a coefficient depending upon the specific heat of air and the caloric of elasticity of its included vapour, its arithmetical value being .01149, or the equivalent vulgar fraction $\frac{1}{87}$. In the paper to which I have already referred, corrections are given for the influence on the specific heat of air, of the fluctuations of the barometer, and the moisture present in the atmosphere. These corrections are, I believe, deduced from correct principles, and should be resorted to when extreme precision is desirable. Experience, however, and a careful consideration of the subject, have satisfied me that they are, generally speaking, in their effects, much too insignificant to be objects of attention to the practical meteorologist.

The first and most obvious method of verification which presented itself to my mind, was the comparison of my formula with recorded cotemporaneous observations on the temperature of air, that shown by a moist-bulb hygrometer, and the actual dew-point. I have, however, unfortunately been able to meet but few at all suited to my purpose. Those in which $t - t'$ is small, and this is generally the case in the few registers to which I have had access, cannot, as we have already seen, serve for deciding the value of any formula. In the first report, indeed, of the British Association for the Promotion of Science, page 50, mention is made of a register of observations kept in the East Indies, which, as belonging to high temperatures, would necessarily exhibit great depressions, and would therefore be valuable as a standard of comparison; but I have in vain searched for the Calcutta Journal, " Gleanings in Science," in which they are said to be contained. In fact, the only observations I have been able to procure, adapted to my purpose, and made, apparently at least, with the necessary precision, are those adduced in the article ' Hygrometry' óf Sir David Brewster's Encyclopædia, and there made, by the author of the article, the basis of a calculation for investigating the constants of a tentative formula for connecting the indications of the wet-bulb hygrometer with the dew-point. They are but two in number, and are comprehended in the following table, in which the numbers in the first column represent the temperatures of air, those in the second the corresponding indications of the hygrometer, those in the third the depressions, those in the

fourth the pressures, and those in the fifth the dew-points experimentally determined by the method of Dalton :—

(1)	(2)	(3)	(4)	(5)	(6)
t	t'	d	p	t'' ob.	t'' calc.
67.2 ...	52 ...	15.2 ...	29.75 ...	35.7 ...	35.6
56.4 ...	49.5 ...	6.9 ...	30.02 ...	39.5 ...	42.4

The numbers in column (6) are the dew-points calculated by my formula; and while there is an almost exact correspondence between the first and the result of experiment, the second, it will be seen, is higher than the observed temperature of deposition by nearly three degrees. There is here, however, obviously some mistake. It is impossible that, with the recorded temperatures of air and hygrometer, the dew-point could have been so low ; and this conclusion I do not at present draw from my theoretical views, for that would be to subject myself to the imputation of arguing in a circle ; but from the following observation, made by me with great care on the 22d of March :—

t	t'	d	t'' ob.
56	50	6	44

Here the temperatures t and t' differ from those taken from the Encyclopædia only by about half a degree ; and nevertheless the observed dew-point 44 is higher than 39.5 by 4.5 degrees. From these observations, therefore, I am, I conceive, entitled to conclude—1st, that the series in which the depression amounts to $15°.2$, being in exact accordance with my formula, lends it some degree of support; and 2dly, that my method cannot be considered as impugned by the other series, inasmuch as *this* is in some particular manifestly incorrect. But it is time to enter upon the experimental tests to which I have resorted.

If air, in reference to which t, t' and t'' have been accurately noted, be raised to any elevated temperature, and the observation be repeated in the heated air, as far as respects t and t', we will have two* separate sets of observations, from which to calculate the point of deposition ; and as the amount of moisture in the air is not altered by the augmentation of temperature it has experienced, both calculations, provided our formula be correct, should give precisely the same result; *i. e.* the dew-point in the first instance determined by observation. Such is the principle of the test experiments which I first performed. The air was heated by urging it in a continued stream, by means of a double bellows, through the worm of a small still—such as are for sale in the Opticians' shops—the worm-tub being filled with water of the desired temperature ; and, in order to the necessary observations, in a glass tube connected by a cork with the upper extremity of the worm, a couple of small thermometers were

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Any number of observations, having reference to the same dew-point, may, it is obvious, be thus obtained.

placed, their bulbs being separated by about a quarter of an inch, and that of the instrument occupying the higher position being invested with a tunic of muslin kept constantly moist with water. The blast was steadily maintained until the thermometers ceased to rise, and the temperature of each was then accurately noted, the eye being assisted by a lens. Tables 1, 2, 3, and 4 exhibit the results of four distinct series of experiments thus conducted :—

TABLE 1.
February 8, 1835, 11 o'clock, A.M.

	t	t'	d	p	t'' ob.	t'' calc.	Diff.
1	49.6	44.7	4.9	29.6	40	39.02	—.98
2	88.5	62	26.5	29.6	40	39.18	—.82
3	80.5	59	21.5	29.6	40	39.27	—.73

TABLE 2.
February 9, 1835 : 11 o'clock, A.M.

	t	t'	d	p	t'' ob.	t'' calc.	Diff.
1	47.2	42.5	4.7	30.02	38	36.58	—1.42
2	76	57.5	18.5	30.02	38	40.44	+2.44

TABLE 3.
March 4, 1835 : 11 o'clock, A.M.

	t	t'	d	p	t'' ob.	t'' calc.	Diff.
1	48.3	43	5.3	29.76	37.5	36.41	—1.09
2	96	64	32	29.76	37.5	36.37	—1.13
3	91	62.5	28.5	29.76	37.5	37.66	+ .16
4	75	56	18	29.76	37.5	38.26	+ .76

TABLE 4.
March 25, 1835 : 11 o'clock, A.M.

	t	t'	d	p	t'' ob.	t'' calc.	Diff.
1	51.3	45.5	5.8	30.7	38.5	38.61	+ .11
2	82	59	23	30.7	38.5	36.7	—1.8

The results exhibited in the preceding tables will, I believe, be considered by many as going far towards establishing the accuracy of my theoretical views. Although the depressions vary from $4°.7$ to $28°.5$, the differences between the observed dew-points and those deduced from the formula, are certainly not greater than what may fairly be ascribed to unavoidable inaccuracy of observation. But for the purpose of putting this matter in a still clearer point of view, I have calculated a number of values of m, the constant of our formula, from the preceding observations. This was easily done ; for as all the observations in the same table refer to air in the same hygrometical state, each series should give the same dew-point, and the expression $f' - m d \times \dfrac{p}{30}$ must have in reference to them a constant value. $f' - m d \times \dfrac{p}{30}$ for one must therefore be qeual to $F' - m D \times \dfrac{p}{30}$ for any other—an equation from which we deduce $m = \dfrac{F'-f'}{D-d} \times \dfrac{30}{p}$. The application of this method gives us the following values of m :—

TABLE 1.

(1 and 2)	(1 and 3)	(2 and 3)
$m = .01155$ ...	.01185 ...	.01075

TABLE 2.

(1 and 2)
$m = .01489$

TABLE 3.

(1 and 2)	(1 and 3)	(1 and 4)	(2 and 3)	(2 and 4)	(3 and 4)
$m = .01137$ ...	.01187 ...	.01309 ...	.00825 ...	.00976 ..	.01045

TABLE 4.

(1 and 2)
$m = .00967$

If the mean of all these values of *m* be taken, it will be found to be .01122, or the equivalent vulgar fraction $\frac{1}{89}$, an approximation to the coefficient $\frac{1}{87}$ employed in the formula, which, under all the circumstances, cannot but be considered as remarkably close. Indeed the difference, which is less than 3 in the fourth place of decimals, is so small, that they may be substituted indiscriminately for each other without the occurrence, at least in ordinary cases, of sensible error. Had values of *m* been calculated from the comparison alone of the first series of observations in each table with the subsequent ones, the mean, it is worthy of remark, would be .01156, or almost exactly $\frac{1}{87}$; and as, for such observations, $F'-f'$ and $D-d$ are necessarily greatest, they are best calculated to afford correct results, since any error of experiment would obviously, in their case, exercise the least influence.

The next test experiments performed were suggested by the formula itself. If $f''=f'-\frac{d}{87}\times\frac{p}{30}$, and f'' be supposed equal to o, a condition which can only be fulfilled in perfectly dry air, $f'=\frac{d}{87}\times\frac{p}{30}$, an equation from which we deduce $d=87f'\times\frac{30}{p}$. Hence, by determining experimentally the depression of the hygrometer in perfectly dry air, we will be able to pronounce upon the validity of the general method under discussion.

The first attempts for determining values of *d* experimentally, consisted in suspending a pair of thermometers, one of which had its bulb moistened, in a close corked bottle, the bottom of which was covered with a stratum of oil of vitriol; but this method was soon abandoned, as the depressions it afforded were, on an average, one-fifth less than they should be according to the formula. In fact the extreme depression could not be expected here, for it is obvious that the air, in contact with the bulb of the moist thermometer, is never perfectly dry except at the very commencement of the experiment.

The next contrivance to which I resorted was as follows. A bag of India rubber cloth, furnished with a cap and stop-cock, was inflated by a bellows, and then connected, by means of a caoutchane collar, to a glass tube, traversing a cork fitted to the tubulure of the lower bottle of a Noothe's apparatus. The middle bottle of the apparatus was next filled, $\frac{2}{3}$rds, with oil of vitriol, and the pair of thermometers last described being introduced into the axis of a small tube, perforating a cork fitted to the upper opening of this bottle, a stream of air was forced by pressing on the caoutchouc bag, through the oil of vitriol, and, of course, over the thermometers; and as soon as the instrument with moistened bulb ceased to fall, the temperatures of both were noted. The following table comprehends the results of five experiments thus performed :—

	t	t	p	d ob.	d calc.	Diff.
March 11	48.5	31.5	29.37	17	17.4	− .4
15	50.5	33.5	30.00	17	18.2	−1.2
20	54.5	35.5	30.25	19	19.4	− .4
21	57.5	37.5	30.27	20	20.8	− .8
22	54.5	36	30.35	18.5	19.7	−1.2

Now, as in all these instances, the observed depression differs from the true ; this difference, though small, being always on the same side, must be ascribed either to the co-efficient m being assumed too great, or to the method of experiment employed not being calculated to afford the extreme depression. That this latter was the real cause of the discrepancy I was disposed to believe, from having observed that when the hygrometer, in the course of an experiment, became stationary, it could be made to sink a little further by pressing with great force upon the bag of air. In fact, this observation rendered it probable that the tube, between the lower and middle bottle of the Nooth, did not afford sufficient air-way; and that, therefore, there was not a sufficient current from behind to propel forward, and immediately remove, from contact with the moistened bulb, the air which had become saturated with its humidity. To bring this conjecture to the criterion of experiment, it was obviously necessary to operate so, that while the air underwent perfect desiccation, it was, at the same time, made to pass over the thermometers in a strong and continuous current; and, after some trials, I found that both objects were secured by substituting for the Nooth a series of three Wolfe's bottles, containing oil of vitriol, and connected, as in the process for preparing the water of ammonia, by glass tubes and caoutchouc collars, the bag of air being attached to a tube passing to the bottom of the first bottle, and the thermometers being placed in the axis of a tube perforating a cork inserted into one of the tubulures of the last bottle. The experiments recorded in the following table were made with this apparatus.

	t	t'	p	d ob.	d calc.	Diff.
March 26	51	33.5	30.55	17.5	17.94	+.44
27	53	34.5	30.35	18.5	17.73	−.77
28	52	34	30.21	18	17.62	−.38
29	51	33	30.05	18	17.97	−.03
30	52	33.4	29.75	18.6	18.37	−.23
31	53	34.3	29.50	18.7	19.14	+.44
April 1	56.5	35.8	29.70	20.7	20.04	−.66
2	58	37	29.72	21	20.88	−.12
3	58.2	37	29.77	21.2	20.84	−.36
4	58	37	30.03	21	20.68	−.32
5	58	37	30.15	21	20.59	−.41
6	59	37.5	30.25	21.5	20.88	−.62
7	59	38	30.26	21	21.24	+.24
8	61	38.7	30.21	22.3	21.80	−.50
10	58.3	37.7	30.35	20.6	20.96	+.36
11	58	37.5	30.45	20.5	20.75	+.35
12	56.3	36.5	30.30	19.8	20.12	+.32
13	57.5	37	30.20	20.5	20.55	+.05
14	57.5	37	30.15	20.5	20.59	+.09

Of the nineteen observations of depression in dry air registered in the preceding table, eleven are greater, and eight less than the calculated results. The mean of the plus errors of the formula is, .28, and of the minus errors, .4 of a degree ; so that .28 — .40 = — .12 of a degree is the mean difference deducible from the whole between experiment and calculation. A closer approximation between them than this could not, I think, be anticipated, even upon the hypothesis of the strict accuracy of the formula. I may also observe that if by means of the equation $f' = m\,d \times \frac{p}{30}$, which, as we have already seen, belongs to perfectly dry air, we deduce from the preceding tables 19 values of m, the mean of all will be found almost accurately equal to $\frac{1}{87}$, a result the more entitled to confidence inasmuch as the mean pressure for the 19 experiments being but very little over 30, and the air being perfectly dry, neither of the corrections which I investigated in my former paper require to be applied.

If from the experiments already detailed I were to draw the conclusion that the equation $f'' = f' - \frac{d}{87} \times \frac{p}{30}$ will afford the dew-point with a degree of accuracy far surpassing ordinary hygrometrical observations, I would, probably, have the concurrence of most of my readers. The evidence adduced in support of the formula appears, at least to me, ample and satisfactory. For the purpose, however, of dispelling any doubts of its accuracy which may exist in the minds of others, I undertook another series of test experiments, to the description of which I shall now proceed.

The most direct method of testing our formula consists, as has been already observed, in comparing its results with dew-points experimentally determined. In order, however, that this criterion be decisive, it is not only necessary that the depressions be considerable in amount, but also, as is obvious, that the dew-points be accurately known. Now the registers to which I have had access do not perfectly satisfy either of these conditions, the depressions being generally small, and the observations made with an instrument—Daniell's hygrometer, the difficulty of observing with which is universally admitted. In reflecting on this matter it occurred to me that both difficulties might be evaded in the following simple manner. Let air, saturated with moisture, and whose temperature is, therefore, necessarily its dew-point, be heated, and let the temperature of the heated air be taken, as also that shewn by a moist bulb hygrometer, subjected to the action of a current of it. Let, then, by the application of the formula, the dew-point, belonging to the two latter observations, be calculated, and from a comparison of it with the original temperature of the air when saturated with humidity, we will be enabled to pronounce with confidence upon the value of our method.

In the experiments which I performed on this plan the air was saturated with moisture, by forcing it from a bellows through a succession of four Wolfe's bottles, connected in the usual way so as to cause the air to pass in each bottle through about two inches of water, and the air thus saturated was heated by being made to pass through a coil of copper tubing, immersed in a tub of warm water, the thermometer and hygrometer being placed with their bulbs within a quarter of an inch of each other in a narrow glass tube, attached to the farther extremity of the copper worm. The following are the results thus obtained :—

	t	t'	d	p	t ob.	t calc.	Diff.
April 17, 1835, 11 o'clock, A.M.	78	62.2	15.8	30.30	51.3	50.47	—. 83
	76	61.5	14.5	30.30	51.3	50.26	—1.04
	73	60.3	12.7	30.30	51.3	51.58	+ .28
	72	60	12	30.30	51.3	50.81	— .49
	69	58.6	10.4	30.30	51.3	50.40	— .90
April 18, 1835, 11 o'clock, A.M.	90.5	67	23.5	30.15	50.8	50.17	— .63
	82.2	64.3	17.9	30.15	50.9	51	— .10
	79	62	16.4	30.15	50.9	50.23	— .67
	71.7	60	11.7	30.15	51.2	50.66	— .54
	69	58.9	10.1	30.15	51.5	50.70	— .80
April 20, 1835, 11 o'clock, A.M.	92	69	23	30.42	54.1	54.40	+ .30
	83	65.8	17.2	30.42	54.5	54.36	— .14
	76	63.3	12.7	30.42	54.9	54.54	— .36
	68	60.3	7.7	30.42	55	54.74	— .26
April 21, 1835, 11 o'clock, A.M.	98.5	71.5	27	30.36	55.5	55.51	+ .01
	84.6	67	17.6	30.36	56	55.79	— .21
	77.5	64.5	13	30.36	56.3	55.97	— .33
	81	62.2	8.8	30.36	56.5	56.18	— .32
April 22, 1835, 11 o'clock, A.M.	83	66.5	16.5	30.51	56.8	55.87	— .93
	77	65	12	30.51	57.2	57.23	+ .03
	71.3	63	8.3	30.51	57.5	57.47	— .03
April 23, 1835, 11 o'clock, A.M.	91.8	68.6	23.2	30.51	54.1	53.70	— .40
	75.2	63.2	12	30.51	55	54 94	— .06
	72	62	10	30.51	55.1	54.98	— .12
							— .35=mean.

By a glance at the preceding table, which includes twenty-four distinct observations, we will perceive, 1st. that in the case of seven of them, the observed and calculated dew-points are almost coincident; 2d. that the difference in no instance exceeds, and in but a single instance reaches, one degree; and 3d. that the mean difference, deducible from the whole, is but .35, or about one-third of a degree Fahrenheit. It will also be noted that the difference is negative, or that the mean calculated dew-point is lower than the observed, and not vice versâ. If we were justified in considering this latter result as any thing more than accidental, it might certainly be urged as an argument against the strict accuracy either of our experiments, or our theoretical views ; for the corrections for the influence of pressure and aqueous vapour on the specific heat of air being neglected in the preceding calculations, the

calculated dew-points, instead of being lower, should be higher than the truth. In order, in fact, to account for the discrepancy in question, supposing it to be well established, it would be necessary to conclude either that m, the co-efficient of our hygrometric formula, is assumed somewhat too great, or that the observed depressions are a little too small. The first, I believe, to be the true solution, and am, at present disposed to consider m as more correctly represented by the fraction $\frac{1}{88}$ than $\frac{1}{87}$. This point, however, I have not as yet been able fully to satisfy myself upon, nor can the more exact determination of the value of the constant be considered a matter of much practical importance, since the formula, in its present state, conducts, as we have seen, to results which harmonize admirably with each other and with observation.

I shall conclude by subjoining a couple of tables, by the aid of which the application of my formula $f'' = f' - \frac{d}{87} \times \frac{p}{30}$, to the determination of the dew-point, is greatly facilitated. Table (A), which I have taken from the Edinburgh Encyclopædia, article hygrometry, gives the elastic force of the vapour of water for every degree Fahrenheit between 0° and 100° inclusive. Table (B) gives $\frac{d}{87 \times 30}$ for every value of d between 1 and 10. This quotient, as is obvious from a glance at the formula, is, in calculating an observation, to be multiplied by p the existing pressure, and the product, when deducted from f', as given by table (A), will afford f'', or the tension of vapour, at the dew-point. Should the depression exceed 10° the value of $\frac{d}{87 \times 30}$ may still be got from table (B) by addition. Thus if $d = 13°$, $\frac{d}{87 \times 30} =$ 00383 + .00114 = .00497.

TABLE (A.)

t	f	t	f	t	f	t	f	t	f
0	·06121	21	·13408	41	.27376	61	.54089	81	1.03350
1	·06359	22	·13906	42	.28346	62	.55913	82	1.06656
2	·06605	23	.14421	43	.29348	63	.57795	83	1.10058
3	·06861	24	.14954	44	.30384	64	.59715	84	1.13559
4	·07126	25	.15506	45	.31453	65	.61734	85	1.17161
5	·07401	26	.16076	46	.32557	66	.63795	86	1.20867
6	·07685	27	·16667	47	.33684	67	.65919	87	1.24680
7	·07980	28	.17277	48	.34875	68	.68108	88	1.28602
8	·08286	29	·17908	49	.36090	69	.70364	89	1.32636
9	·08603	30	18561	50	·37345	70	.72688	90	1.36785
10	·08931	31	·19237	51	·38640	71	.75083	91	1 41059
11	·09270	32	·19934	52	·39977	72	.77551	92	1.45438
12	·09622	33	·20658	53	·41356	73	.80092	93	1.49948
13	·09987	34	·21404	54	·42779	74	.82710	94	1.54585
14	·10364	35	·22175	55	·44249	75	.85407	95	1·59352
15	·10755	36	·22972	56	45764	76	.88184	96	1 64251
16	·11160	37	·23796	57	·47328	77	.91042	97	1·69286
17	·11579	38	·24647	58	·48940	78	.93987	98	1.74461
18	.12013	39	·25527	59	·50604	79	.97017	99	1.79778
19	.12462	40	·26436	60	·52320	80	1.00137	100	1.85241
20	.12927								

TABLE (B.)

d	$\dfrac{d}{87 \times 30}$	d	$\dfrac{d}{87 \times 30}$	d	$\dfrac{d}{87 \times 30}$	d	$\dfrac{d}{87 \times 30}$	d	$\dfrac{d}{87 \times 30}$
.1	.00003	2.1	.00080	4.1	00157	6.1	0033	8.1	.00310
.2	.00007	2.2	.00084	4.2	00160	6.2	.00237	8.2	.00313
.3	.00011	2.3	.00087	4.3	00164	6.3	.00241	8 3	.00317
.4	.00015	2.4	.00091	4.4	00168	6.4	.00245	8.4	.00321
.5	.00019	2.5	.00095	4.5	00172	6.5	.00248	8.5	.00325
.6	.00022	2.6	.00099	4.6	00176	6.6	.00252	8.6	.00329
.7	.00026	2 7	.00103	4.7	00180	6.7	.00256	8.7	.00333
.8	.00030	2.8	.00107	4.8	00183	6.8	.00260	8.8	.00337
.9	.00034	2.9	.00111	4.9	00187	6.9	.00264	8.9	.00340
1	.00038	3	.00114	5	00191	7	.00268	9	.00344
1.1	.00042	3.1	.00118	5.1	00195	7.1	.00271	9.1	.00348
1.2	.00045	3.2	.00122	5.2	00199	7.2	.00275	9.2	.00352
1.3	.00049	3.3	.00126	5.3	00202	7.3	.00279	9.3	.00356
1.4	.00053	3.4	.00130	5.4	00206	7.4	.00283	9.4	.00360
1 5	.00057	3.5	.00134	5.5	00210	7.5	.00287	9.5	.00363
1 6	.00061	3.6	.00137	5.6	00214	7.6	.00291	9.6	.00367
1 7	.00065	3.7	.00141	5.7	00218	7.7	.00294	9.7	.00371
1 8	.00068	3.8	.00145	5.8	00222	7.8	.00298	9.8	.00375
1.9	.00072	3.9	.00149	5.9	00225	7.9	.00302	9.9	.00379
2	.00076	4	.00153	6	00229	8	.00306	10	00383